reduction in baffle area may be effectively used for additional baffles, thus increasing the minimum damping ratio value with no appreciable increase in baffle weight. The liquid natural frequencies are somewhat dependent on both the ring width and the ring baffle spacing. The first liquid resonant frequency increases over that of a bare wall tank only for baffle positions in the immediate vicinity of the liquid surface, and decreases below that of a bare wall tank for more deeply submerged ring baffles.

For a vehicle in which the first liquid resonant frequency is not critically coupled with the control system, ring baffle damping is therefore probably the most efficient means of introducing liquid damping into the propellant tanks. If, however, the liquid resonant frequencies are critical and must be shifted by rather significant amounts, other baffle systems should be considered.

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Gyroscopic Attitude Stabilization

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Introduction

In an earlier investigation,[1] the feasibility of gyroscopic attitude stabilization of satellites and space vehicles was examined and found to be realizable. The stabilization was attained through an internally moving disk gyro, but gravitational effects were neglected. Thomson[2] has studied stabilization, including gravitational effects, but required the spinning of the entire satellite. Kane and Sobala,[3] also including gravitational effects, have demonstrated the practicability of obtaining stabilization through internally moving particles. The purpose of the present investigation is to explore further, by including gravitational effects, the concept of stabilization through an internally moving disk gyro.

Governing Equations

The system to be studied is shown in Fig. 1 where V represents a rotationally symmetric satellite with L_3 being the axis of symmetry. D represents a circular disk gyro whose axis is L_3 and whose mass center C coincides with the mass center of V. L_1 and L_2 are axes in the plane of D, which form with L_3 a dextral set. Because of the symmetry, these axes are centroidal principal inertia axes, although L_1 and L_2 are not fixed in either V or D. Finally, parallel to each axis, there is a unit vector $\mathbf{n}_i$ ($i = 1, 2, 3$).

The system is considered to be in a motion in which C describes a circular orbit. Let $\dot\theta$ represent the angular rate of change, in inertial space, of a radial line drawn from O, the orbit center, to C. By considering a configuration in which L_3 is normal to the orbit plane and L_1 is parallel to OC in the direction OC, three angles, α, β, and γ, may be introduced and defined by successive dextral rotations about the axes

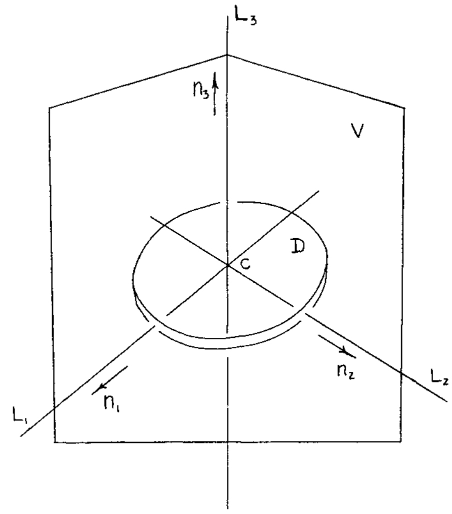

Fig. 1 Satellite with disk gyro.

L_1, L_2, and L_3. The angular velocity of V in inertial space, in which the orbit is taken to be fixed, is then given by

$$\boldsymbol{\omega}_V = (\dot\alpha \cos\beta - \dot\theta \sin\beta \cos\alpha)\mathbf{n}_1 + (\dot\theta \sin\alpha + \dot\beta)\mathbf{n}_2 + (\dot\gamma + \dot\alpha \sin\beta + \dot\theta \cos\alpha \cos\beta)\mathbf{n}_3 \quad (1)$$

Regarding D as having an angular speed Ω with respect to V, the corresponding angular velocity of D is

$$\boldsymbol{\omega}_D = \boldsymbol{\omega}_V + \Omega \mathbf{n}_3 \quad (2)$$

The kinetic energy of the system is then

$$K = (\tfrac{1}{2})[mr^2\dot\theta^2 + I_1\omega_1^2 + I_2\omega_2^2 + (I_3 - 2I_D)\omega_3^2 + 2I_D(\Omega + \omega_3)^2] \quad (3)$$

where r is the orbit radius OC, m is the combined mass of V and D, I_i ($i = 1, 2, 3$) are the combined moments of inertia of V and D with respect to C about the L_i axes, I_D is the moment of inertia of D with respect to C about the L_1 axis, and ω_i are the $\mathbf{n}_i$ measure numbers of $\boldsymbol{\omega}_V$, obtainable from Eq. (1).

The gravitational forces, which have been investigated by Nidey[4] and others, may be represented by a single force passing through C together wth a couple of torque $\mathbf{T}$ given by

$$\mathbf{T} = 3\dot\theta^2(I_1 - I_3)\sin\beta \cos\beta \mathbf{n}_2 \quad (4)$$

By considering α, β, and γ as generalized coordinates in the sense of Lagrangian mechanics, Eqs. (4) and (1) lead to the generalized forces

$$F_\alpha = F_\gamma = 0 \qquad F_\beta = 3\dot\theta^2(I_1 - I_3)\sin\beta \cos\beta \quad (5)$$

The governing equations of motion are then obtained through Eqs. (3) and (5) and are of the form

$$(d/dt)(\partial K/\partial \dot\alpha) - \partial K/\partial \alpha = F_\alpha \quad (6)$$

If, for the purposes of stability considerations, α and β are considered small, these governing equations may be written

$$\ddot\alpha - c_1\dot\beta + c_2\alpha = 0 \quad (7)$$

$$\ddot\beta + c_1\dot\alpha - c_3\beta = 0 \quad (8)$$

$$(\dot\theta + \dot\gamma)I_3 + 2I_D\Omega = \text{const} \quad (9)$$

where the coefficients c_1, c_2, and c_3 are given by

$$c_1 = [2 - (I_3/I_1)]\dot\theta - (I_3/I_1)\dot\gamma - 2\Omega(I_D/I_1) \quad (10)$$

$$c_2 = -(I_3/I_1)\dot\gamma\dot\theta + [1 - (I_3/I_1)]\dot\theta^2 - 2\Omega\dot\theta(I_D/I_1) \quad (11)$$

$$c_3 = -(I_3/I_1)\dot\gamma\dot\theta + 4[1 - (I_3/I_1)]\dot\theta - 2\Omega\dot\theta(I_D/I_1) \quad (12)$$

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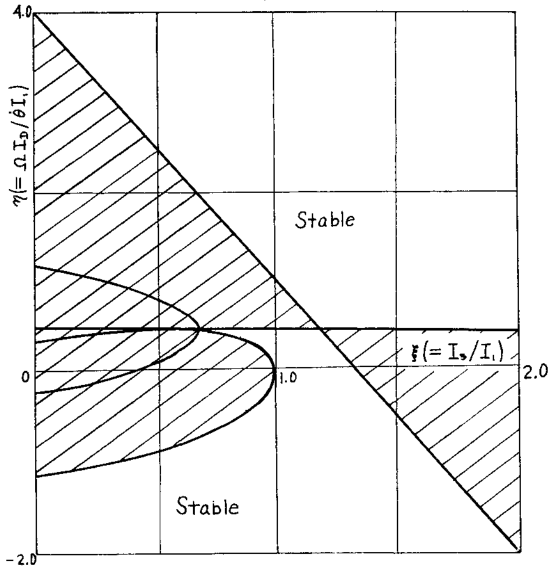

Fig. 2 Stability regions.

Stability Considerations

If Ω is constant, Eq. (9) shows that $\dot{\gamma}$ is also constant, and then the coefficients c_1, c_2, and c_3 are constant. Eliminating β between Eqs. (7) and (8) then leads to the differential equation

$$d^4\alpha/dt^4 + (c_1^2 - c_2 - c_3)d^2\alpha/dt^2 + c_2c_3\alpha = 0 \quad (13)$$

Thomson[2] has shown that solutions of this type of equation will remain small if the coefficients satisfy the inequalities

$$c_1^2 - c_2 - c_3 > 0 \quad (14)$$

$$c_2c_3 > 0 \quad (15)$$

$$(c_1^2 - c_2 - c_3)^2 - 4c_2c_3 > 0 \quad (16)$$

These relations thus represent the stability criteria for a satellite with a constant-speed disk gyro.

As a specific example, consider the case when $\dot{\gamma}$ is exactly equal to the negative of $\dot{\theta}$. This is a situation in which V essentially maintains a fixed attitude in inertial space. Under these conditions, the inequalities (14–16) may be written

$$3\xi - 1 + 4\eta(\eta - 1) > 0 \quad (17)$$

$$4 - 3\xi + 2\eta(2\eta - 5 + 3\xi) > 0 \quad (18)$$

$$9\xi^2 + (6 - 48\eta + 24\eta^2)\xi - 15 + 48\eta - 8\eta^2 - 32\eta^3 + 16\eta^4 > 0 \quad (19)$$

where ξ and η are dimensionless parameters given by

$$\xi = I_3/I_1 \qquad \eta = \Omega I_D/\dot{\theta}I_1 \quad (20)$$

The relations (17–19) determine regions of stability on an ξ-η diagram. These are shown in Fig. 2. Since ξ is essentially a function of the satellite geometry, and η is proportional to Ω, the diagram provides the gyro speed requirements necessary for maintaining stability for the various satellite shapes.

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Optimization of Random Satellite Systems through the Use of Integer Programing Techniques

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A SATELLITE communication system that consists of randomly spaced satellites in circular orbit seems to be a likely choice for future satellite communications. A digital computer program has been developed to obtain the coverage given to any pair of ground stations by a satellite at any altitude and any inclination angle. The program requires only the latitude and longitude of the ground station, altitude, and inclination angle of the satellite, and ground station minimum antenna elevation angle; it can be utilized for both active and passive satellite systems. The assumption of a random satellite system permits some interesting manipulation, which allows us to obtain the optimum mix of satellites in inclination angle and altitude to satisfy specified communication requirements at minimum cost. For the randomness assumption to be correct, satellites must have random orbits for a given inclination, as well as be randomly phased with respect to each other within a given orbit.

Geometry of the Problem

For any two ground stations to be able to communicate, the same satellite must be visible to each. The region of communication for a particular ground station is defined by the antenna elevation angle B, the satellite orbit altitude H, and the latitude S and longitude M of the ground station. The ground antenna can rotate 360° in azimuth and (180°–2B) in elevation; it can see everything that is at least B° above the horizon (line-of-sight). The region of communication is the area formed by the intersection of the cone generated by the antenna with the sphere at altitude H. For circular orbits, the intersection of the antenna cone with the orbit sphere defines a circle, which can be radially projected to the earth, so that the region of communication can be defined in terms of latitude and longitude. Equation (1) defines the longitude boundaries of the region of communication for a specified latitude.[1,2]

$$L = M \pm \cos^{-1}\left[\frac{\sin A - \sin P\cos(90 - S)}{\cos P\sin(90 - S)}\right] \quad (1)$$

where $A = B + \sin^{-1}[R\cos B/(R + H)]$ and L is the longitude of the boundary of the region of communication for a specified latitude P. By next defining a time history of the ground track trajectory of a satellite at a particular altitude H and orbit inclination Q, we can tell what time a satellite will be in the region of communication for any ground station[3]:

$$X = \sin^{-1}(\sin Q\sin\omega t) \quad (2)$$

$$V = \tan^{-1}(\cos Q\tan\omega t) \quad (3)$$

$$Y = Y_0 + V - Et \quad (4)$$

where X and Y are latitude and longitude of the ground track, V the longitude change caused by orbital motion in the absence of earth rotation, E the earth angular rate, and ω the orbital angular velocity. Probabilities of visibility for the links are obtained by indexing the starting point of satellite ground track around the equator and averaging the percentage of time in view obtained at each position.

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